MTH441 HW 4

1. Let k, n ≥ 1 be integers and T : Rk → Rn be a linear function. That is 1. T(x + y) = T(x) + T(y) for all x, y ∈ Rk and 2. T(rx) = rT(x) for all r ∈ R and x ∈ Rk

a. Show that T is continuous on Rk. [From Linear Algebra, you know the general form of a linear function on Rk. Using that knowledge this should just be an “obviously continuous” kind of proof.]

b. Show that T is uniformly continuous on Rk. Notice that Rk is not compact.

1. Let (an) 🡪 a ∈ Rk be a sequence

Since (an) converges to a, |an – a| 🡪 0 as n 🡪 ∞

Scratch: f(an) 🡪 f(a) goal

|f(an) – f(a)| 🡪 0

|f(an – a)| 🡪 0

f(0k) = 0n (by linear function)

Lim n🡪∞ (an – a) = 0k (convergent sequence)

So Lim n🡪∞ f(an – a) = 0n

So Lim n🡪∞ |f(an – a)| = 0 (linear function)

So Lim n🡪∞ |f(an) – f(a)| = 0 (linear function)

So Lim n🡪∞ |f(an)| = a